

A variational method for probing extreme events in turbulent dynamical systems

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Prediction of extreme events for chaotic systems with intrinsically high-dimensional attractors is a formidable problem throughout science and engineering. These are especially challenging issues when real-time prediction is needed, such as, for example, in weather forecasting or prediction of extreme nonlinear waves. Thus, a major challenge in contemporary data-driven modeling of dynamical systems is the computation of low-energy patterns or signals, which systematically precede the occurrence of these extreme transient responses. Here, we propose a variational framework for probing conditions that trigger intermittent extreme events in high-dimensional nonlinear dynamical systems. These algorithms exploit in a combined manner some basic physical properties of the chaotic attractor, as well as, stability properties of the governing equations. Specifically, we seek the triggers as the probabilistically feasible solutions of an appropriately constrained optimization problem, where the function to be maximized is a system observable exhibiting intermittent extreme bursts. We apply the method to two challenging problems: i) the prediction of extreme intermittent bursts of energy dissipation in a prototype turbulent system, the body-forced incompressible Navier–Stokes equation, known as the Kolmogorov flow, and ii) the prediction of extreme events in a dispersive wave model for weak turbulence, the Majda–McLaughlin Tabak (MMT) model. We find that in both cases the intermittent bursts caused by the spontaneous transfer of energy from large scales to the mean flow via nonlinear interactions. The global maximizer of the corresponding variational problem identifies the responsible modes, providing in this way with precursors for the occurrence of extreme dissipation events. We assess the performance of the derived predictors through direct numerical simulations.

