PROBABILISTIC AND DISCRETE METHODS FOR THE COMPUTATIONAL STUDY OF COHERENT BEHAVIOR IN FLOWS

Kathrin Padberg-Gehle

Leuphana Universität Lüneburg, Institute of Mathematics and its Didactics, Scharnhorststr. 1, 21335 Lüneburg, Germany

Transfer operator based numerical schemes have only recently been recognized as powerful tools for analyzing and quantifying transport processes in time-dependent flows. Central to this probabilistic concept are coherent sets [5, 2, 3], mobile regions in phase space that move about with minimal dispersion (see Figure 1). Coherent sets can be efficiently identified via Perron-Frobenius operators (or transfer operators). These linear Markov operators can be approximated within a set-oriented numerical framework and subdominant singular vectors of the resulting stochastic matrices are used to determine the structures of interest.

While transfer operator based schemes require high resolution trajectory data, spatio-temporal clustering algorithms have been proven to be very effective for the extraction of coherent sets directly from sparse and possibly incomplete trajectory data [4, 6, 1]. In particular, a discrete representation of the dynamics in terms of a trajectory network can be used to build a computationally very attractive and flexible approach [7].



Figure 1: Coherent sets in a Bickley jet flow.

In this contribution, we will give an introduction to the probabilistic transfer operator based concepts and the recently proposed discrete trajectory based approaches for the computational study of coherent behavior in flows. We will demonstrate the applicability of these methods in a number of example systems, including turbulent flows.

References

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